Near-Optimal Low Complexity MLSE Equalization

***Abstract*—An iterative Maximum Likelihood Sequence Estimation (MLSE) equalizer (detector) with hard outputs, that has a computational complexity quadratic in the data block and the channel length, is proposed. Its performance is compared to the Viterbi MLSE algorithm that has a computational complexity that is linear in the block length and exponential in the channel memory length. It is shown via computer simulation that the proposed iterative MLSE detector is able to detect Binary Phase-shift Keying (BPSK) signals in systems with significantly larger channel length than what is possible with the Viterbi algorithm, for frequency selective Rayleigh fading channels.**

I. Introduction

For frequency selective channels, MLSE equalizer (detector) based on the Viterbi Algorithm (VA) [1], [2] and the Maximum A Posteriori Probability[1] (MAP) [3] equalizer (detector) are frequently used to mitigate the inter-symbol interference (ISI) caused by the frequency selective channel. Both these methods have a computational complexity linear in the length of the block of data to be detected, but exponential in the length of the channel memory (channel delay spread).

For communication systems with moderate or large bandwidth, the channel memory L is large, and the Viterbi MLSE as well as the MAP detection algorithm have high complexity under those conditions. As an example, in Enhanced Data Rates for GSM Evolution (EDGE), 8PSK modulation is used in GSM channels where L=7 [4] implying that there are 7 taps in the channel impulse response (CIR). Even for a Single Input Single Output (SISO) system in EDGE an optimal MLSE (or MAP) detector based on a Viterbi (or MAP) trellis would require some 8^6 states, clearly beyond what is practical today. Thus the use of Delayed Decision Feedback Equalization (DDFE) [5] is proposed in [4] where the first few taps are equalized using a reduced state trellis, while the ISI caused by the rest of the taps in the CIR is removed by applying feedback based on previous detected symbols. This process causes noise enhancement and there is a corresponding reduction in Bit Error Rate (BER) performance so that the DDFE method is suboptimal.

For Multiple Input Multiple Output (MIMO) systems, joint detection of independent data streams is required, and complexity over the SISO system grows exponentially with the number of transmitting antennas. For that reason, suboptimal methods have been developed with realistic computational complexity, at the cost of BER performance [6][2]. The approach taken in [6] is based on set partitioning [7] and delayed decision feedback equalization (DDFE). If large modulation alphabets with M elements are used, set partitioning is able to reduce complexity significantly by partitioning the modulation constellation. However, set partitioning is not able to reduce the computational complexity due to the channel memory length L.

In [8] an approach is proposed where the Viterbi and Sphere-Constrained methods are combined to perform optimal ML detection, and it is reported that the method has worst case complexity determined by the VA, but is often lower.

Another way of mitigating the problem of high detection complexity for communication systems with large channel memory L is to use Orthogonal Frequency Division Multiplexing (OFDM) modulation [9]. OFDM exploits the orthogonality properties of the Fast Fourier Transform matrix, and is able to perform optimal detection with trivial per symbol complexity regardless of the channel memory length L as long as a cyclic prefix of length greater than L is prepended to the data. However, if the channel memory is large the overhead due to the cyclic prefix becomes significant. Also in a wireless mobile environment OFDM may be vulnerable to Doppler shift, and in general it suffers from a large peak to average power ratio which is undesirable.

In this paper an iterative MLSE detector with hard outputs is proposed with performance that is comparable to the performance of the VA and MAP algorithm, but with computational complexity only quadratic in the data block length N and the channel length L. The formulation is presented for BPSK modulation, but generalization to general M-QAM constellations is possible. The performance of the iterative MLSE detector is compared to that of the Viterbi MLSE algorithm via computer simulation, and it is shown that the new MLSE detector can detect signals in systems with much higher channel lengths than what is currently possible. It is shown that a channel with L =200 and BPSK modulation can be equalized with relative ease using the new iterative equalizer, while for Viterbi MLSE the trellis would have required some 2^{199} states, clearly an impossible task.

The paper is organized as follows. The iterative MLSE detector with hard outputs is presented in Section 2, while in Section 3 it is shown that the computational complexity is quadratic in the data block length N and the channel length L. In Section 4 the raw (uncoded) BER of the new MLSE method is compared to the performance achievable with the Viterbi MLSE detector, for different channel lengths with frequency selective Rayleigh fading. Conclusions are presented in Section 5.

[1] Also referred to as the BCJR algorithm.

[2] The issue of providing soft decision detection to aid the error correction decoder is not dealt with in [6].

II. The iterative MLSE detector with hard outputs

For SISO systems[3], the frequency selective channel model considered here is given by [2], [10]

$$r_k = \sum_{j=0}^{L-1} h_j s_{k-j} + n_k, \tag{1}$$

where s_k denotes the kth symbol in the transmitted sequence of N symbols (the block length) chosen from an alphabet $\mathcal{D}$ containing M complex symbols. r_k is the kth received symbol, n_k is the kth Gaussian noise sample $\mathcal{N}(0, \sigma^2)$, and h_j is the jth coefficient (or tap) of the estimated CIR valid for the data block under consideration [4].

For a block of transmitted symbols of length N, the proposed iterative Maximum Likelihood Sequence Estimator (MLSE) minimizes the cost function [10]

$$\mathcal{L} = \sum_{k=1}^{N} \left| r_k - \sum_{j=0}^{L-1} h_j s_{k-j} \right|^2, \tag{2}$$

to find the most likely transmitted sequence $\mathbf{s} = \{s_1, s_2, \cdots, s_N\}^T$ where ^{T} denotes the transpose operation. The VA is able to solve this problem exactly, with computational complexity linear in N but exponential in L [10].

A. An appropriate Lyapunov function and iterative MLSE detector

The iterative MLSE detector will be formulated here for BPSK, while the generalization to general modulation constellations is possible but not considered in this paper. Equation (2) can be written as

$$\mathcal{L} = -\frac{1}{2}\mathbf{s}^\dagger \mathbf{T}\mathbf{s} - \mathbf{I}^\dagger \mathbf{s}, \tag{3}$$

where $\mathbf{I}$ is a column vector with N elements, $\mathbf{T}$ is a square matrix with N rows and columns, and † implies the Hermitian transpose. $\mathbf{T}$ is symmetric and banded with the width of the band of non-zero elements determined by L, and it is a function of $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_{L-1}\}$, which in turn is a function of the CIR $\mathbf{h} = \{h_0, h_1, \cdots, h_{L-1}\}^T$. $\mathbf{I}$ on the other hand is a function of the observations[4] $\mathbf{r} = \{r_1, r_2, \cdots, r_{N+L-1}\}^T$, and α.

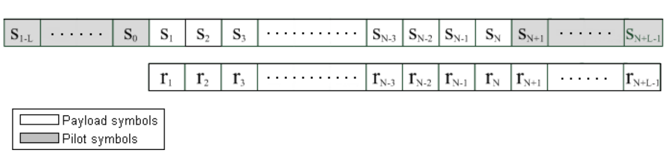

Fig. 1. Transmitted and received data blocks.

[3]The formulation can be generalized to MIMO systems in a straightforward manner

[4]Instead of N observations, $N + L - 1$ observations are used to preserve the multipath information for optimal ISI mitigation

Consider a data block of payload bits of length N, assuming the CIR has L taps and that the block of payload bits are initiated and terminated by L-1 known tail symbols[5], as shown in Fig. 1, and

$$\alpha_k = \sum_{j=0}^{L-k-1} h_j h_{j+k}, \tag{4}$$

with $k = 1, 2, 3, ..., L-1$, $\mathbf{T}$ is given by

$$-4\begin{bmatrix} 0 & \alpha_1 & \alpha_2 & \dots & \alpha_{L-1} & \dots & 0 \\ \alpha_1 & 0 & \alpha_1 & \alpha_2 & \dots & \alpha_{L-1} & \vdots \\ \alpha_2 & \alpha_1 & 0 & \alpha_1 & \ddots & \vdots & \alpha_{L-1} \\ \vdots & \alpha_2 & \alpha_1 & \ddots & \ddots & \alpha_2 & \vdots \\ \alpha_{L-1} & \vdots & \ddots & \ddots & 0 & \alpha_1 & \alpha_2 \\ \vdots & \alpha_{L-1} & \dots & \alpha_2 & \alpha_1 & 0 & \alpha_1 \\ 0 & \dots & \alpha_{L-1} & \dots & \alpha_2 & \alpha_1 & 0 \end{bmatrix} \tag{5}$$

and $\mathbf{I}$ is given by

$$4\begin{bmatrix} r_1 h_0 + \ldots + r_L h_{L-1} - \alpha_1 - \alpha_2 - \ldots - \alpha_{L-1} \\ r_2 h_0 + \ldots + r_{L+1} h_{L-1} - \alpha_2 - \ldots - \alpha_{L-1} \\ r_3 h_0 + \ldots + r_{L+2} h_{L-1} - \ldots - \alpha_{L-1} \\ \vdots \\ r_{L-1} h_0 + \ldots + r_{2L-2} h_{L-1} - \alpha_{L-1} \\ r_L h_0 + \ldots + r_{2L-1} h_{L-1} \\ \vdots \\ r_{N-L+1} h_0 + \ldots + r_N h_{L-1} \\ r_{N-L+2} h_0 + \ldots + r_{N+1} h_{L-1} - \alpha_{L-1} \\ \vdots \\ r_{N-2} h_0 + \ldots + r_{N+L-3} h_{L-1} - \ldots - \alpha_{L-1} \\ r_{N-1} h_0 + \ldots + r_{N+L-2} h_{L-1} - \alpha_2 - \ldots - \alpha_{L-1} \\ r_N h_0 + \ldots + r_{N+L-1} h_{L-1} - \alpha_1 - \alpha_2 - \ldots - \alpha_{L-1} \end{bmatrix} \tag{6}$$

With reference to the function g shown in Fig. 2, in the limit where the gain $\beta \to \infty$, s_k can be written as a function of a variable u_k as

$$s_k = g(\beta u_k). \tag{7}$$

It was shown in [11] that (3) is a Lyapunov function (in the high gain limit where $\beta \to \infty$) for the dynamic system given by

$$\frac{d\mathbf{u}}{dt} = -\frac{\mathbf{u}}{\tau} + \mathbf{T}\mathbf{s} + \mathbf{I}, \tag{8}$$

where τ is an arbitrary (settling) constant and $\mathbf{u} = \{u_1, u_2, \cdots, u_N\}^T$. The dynamical system starting from a zero initial state will move to settle into a steady state denoted $\mathbf{u}^*$ so that $\mathbf{s}^*$ (corresponding to $\mathbf{u}^*$) will minimize the cost function $\mathcal{L}$. $\mathbf{s}^*$ is therefore the MLSE sequence

[5]The transmitted tails are s_{1-L} to s_0 and s_{N+1} to s_{N+L-1} and are equal to 1

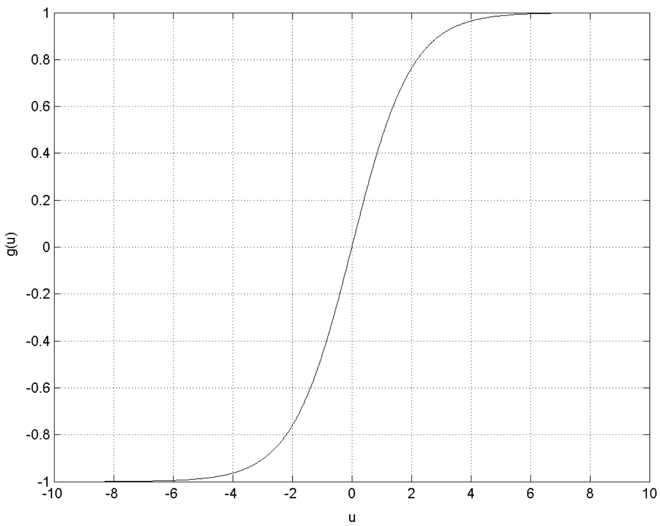

Fig. 2. The sigmoid function $g(u)$.

estimate.

1) The iterative MLSE detector with hard outputs: An iterative solution for (8) is given by

$$\mathbf{u}^{n+1} = \mathbf{T}\mathbf{s}^n + \mathbf{I}$$
$$\mathbf{s}^{n+1} = g(\beta\mathbf{u}^{n+1}) \tag{9}$$

where n indicates the iteration number. Equation (9) represents the proposed iterative MLSE detector. As the system iterates[6], β is updated systematically according to an exponential function[7] to ensure that the system converges to a near-optimal local minimum in the solution space. The β-updates are performed according to the function

$$\beta = 5^{\frac{2(n-Z+1)}{Z}}, \tag{10}$$

where Z indicates the number of iterations. This causes β to start at a near-zero value and to exponentially converge to 1 with each iteration. This, together with asynchronous updates[8], ensure near-optimal sequence estimation. It is also useful to add an extra term[9] to u^{n+1} with each iteration for systems with short CIR lengths ($L < 15$) and for systems with longer CIR lengths at low signal-to-noise ratio (SNR) values. This will cause the system to escape less optimal local minima in the solution space, in order to increase the BER performance. These observations will be tested in Section 4 by examining the BER when the iterative MLSE detector is compared to the Viterbi MLSE detector.

[6]The CIR and the received symbols must be normalized

[7]These values can be store in a lookup table

[8]The update schedule is sequential

[9]The added term was $5s^n$

III. The computational complexity of the proposed iterative MLSE detector

A block of N transmitted symbols $\mathbf{s} = \{s_1, s_2, \cdots, s_N\}$ is transmitted and is to be detected using the MLSE detector given by (9). The iterative MLSE detector requires Z iterations, and in the next section it will be shown that Z may be chosen as 20. Although 20 iterations are used as the norm, it can be adjusted to be as low as 5 for systems with longer CIR lengths ($L > 20$), without a penalty in performance. This is possible due to the effective time diversity provided by the frequency-selective Rayleigh fading channels.

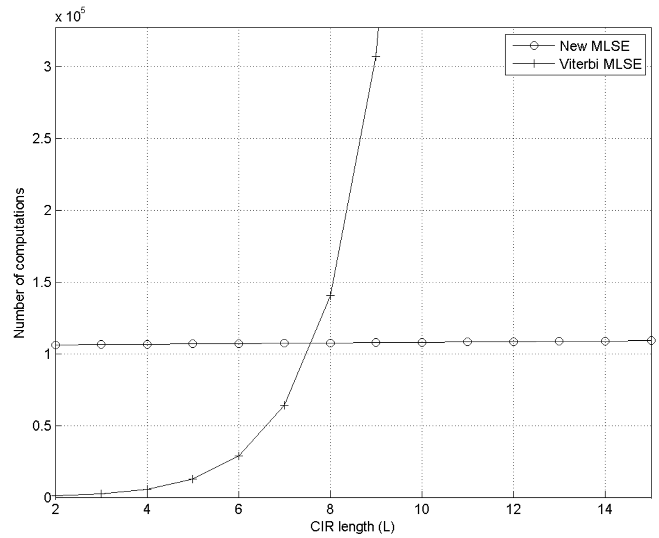

Fig. 3. Computational complexity comparison of the new MLSE detector and the Viterbi MLSE detector.

In general for a modulation alphabet using BPSK, a data block length of N, a CIR length of L, and Z iterations, the computational complexity of the new MLSE detector is $2ZN(N+3) + 4L(N+1) + L^2$ [10]. The Viterbi MLSE detector has a computational complexity $\propto NM^{(L-1)}$ (M=2 for BPSK). Fig. 3 shows the computational complexity comparison of the two detectors, where the data block length was chosen to be 50, with the CIR length from L=2 to L=15, and Z=20 iterations. For SISO systems, where the channel memory length is small (for BPSK the break-even point is at about L=8), the computational complexity of the new MLSE detector is much higher than that of the Viterbi MLSE detector. However for channels with large L, the advantage of having computational complexity per transmitted symbol that is quadratic in N and L rather than exponential in L becomes clear and the reduction in complexity is significant.

[10]For $N >> L$, as is the case in practical systems, the computational complexity can be approximated by $2ZN(N+3)$

IV. Numerical results

In this section, the raw (uncoded) BER for a communication system employing BPSK modulation is compared for the two detectors. The first is the proposed iterative MLSE detector, and the second is the Viterbi MLSE detector. Frequency selective Rayleigh fading[12] channels in burst mode[11] with short CIR lengths and long CIR lengths are investigated separately. In all simulations the nominal CIR settings were chosen as $\mathbf{h} = \{1, 1, \cdots, 1\}$ and normalized so that $\mathbf{h}^T\mathbf{h} = 1$ irrespective of the CIR length, where $\mathbf{h}$ is a column vector and ^{T} denotes the transpose. $L-1$ tail symbols were also added on both sides of the burst as is the case in practical communication systems. Least Squares (LS) channel estimation was used to estimate the CIR in the receiver. This was done so that the effect of imperfect channel estimation is included in the BER results for both algorithms. Additive white Gaussian noise was added in the receiver, the symbol rate, T_s, was set to $3.7\mu s$ and the carrier frequency was 900 MHz.

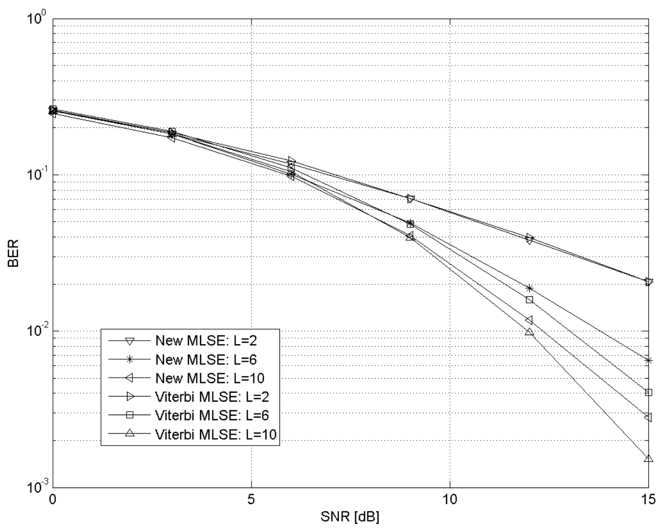

Fig. 4. The BER for CIR lengths 2, 6 and 10 with Rayleigh fading.

First, CIR lengths of 2, 6 and 10 are considered. The data block contained 200 uncoded data payload bits, 20 iterations were used for the new MLSE detector and the mobile speed was set to 50 km/h for all the cases. The number of pilots used for channel estimation was $3L$. The BER was evaluated via computer simulation as shown in Fig. 4 for the Viterbi MLSE detector and the proposed iterative MLSE detector. The BER indicates that the performance of the new MLSE detector is comparable to that of the Viterbi MLSE detector. For low SNR values, the proposed MLSE detector's performance matches that of the Viterbi MLSE detector very closely. However, as the SNR increases, there is a small performance degradation in the proposed MLSE detector.

[11]Frequency hopping is employed so that each burst fades independently

Next, the case of long CIR lengths are considered. These are systems with CIR lengths that are too long for the Viterbi MLSE detector to be applied. Because the computational complexity of the proposed iterative MLSE detector is quadratic in N and L, it can detect BPSK signals in systems with literally hundreds of CIR taps. However, the performance of the new MLSE detector can no longer be compared to that of the Viterbi MLSE detector, as the latter detector cannot be simulated under these conditions.

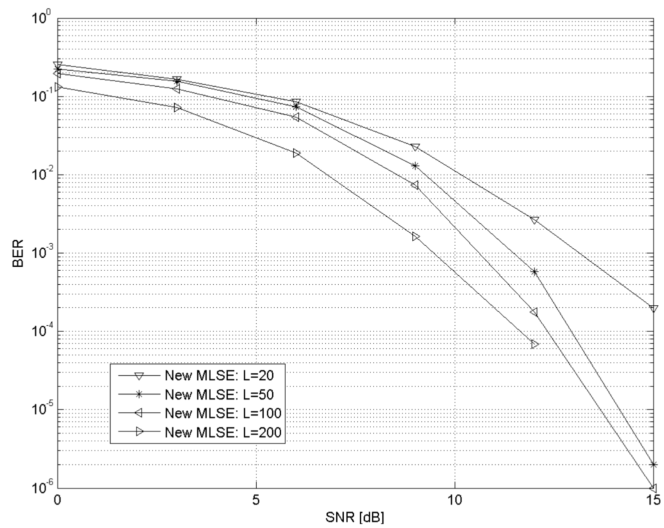

Fig. 5. The BER for CIR lengths 20, 50, 100 and 200 with Rayleigh fading.

We will now consider channels with CIR lengths of 20, 50, 100 and 200, with a data payload block length of 1000, using 20 iterations and a speed of 3 km/h. The number of pilots used for channel estimation was chosen to be 4L. The BER was evaluated via computer simulation as shown in Fig. 5. The results confirm that the new iterative MLSE detector successfully detected the BPSK signals in the long frequency selective fading channels.

V. Conclusions

An iterative MLSE detector with hard outputs was proposed and compared to the Viterbi MLSE detector via computer simulation. Results showed that the proposed MLSE detector produces a slightly worse BER than the Viterbi MLSE detector for channels with short CIR lengths with frequency selective fading. However, because of the computational significance of the new MLSE detector, it is able to detect signals in systems with very large CIR lengths, where the Viterbi MLSE cannot be applied, let alone simulated. It is clear that, based on the results presented, there now exists a general detector for BPSK modulation to detect signals in systems with much longer CIR lengths than what is currently possible. Computational complexity for the new MLSE detector was shown to be $2ZN(N+3) + 4L(N+1) + L^2$, while for the Viterbi MLSE detector the complexity is $\propto NM^{(L-1)}$.

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