Interval fuzzy modelling in fault detection for a class of processes with interval-type parameters

Abstract—**In the paper an application of the interval fuzzy model (INFUMO) in fault detection for a class of processes with uncertain interval-type parameters is presented. A confidence band for the process input-output data is obtained and approximated using a fuzzy model with interval parameters. The approximation is based on partial fuzzy linear programming using l_∞-norm as a measure of the modelling error. Applying high-pass filtering when obtaining the confidence band makes it possible to use arbitrary sets of identification input signals. An application of the INFUMO the in detection of load change in a motor-generator system is presented to demonstrate the benefits of the proposed method.**

I. Introduction

Takagi-Sugeno fuzzy models [16] have been widely used in recent years for building models of nonlinear processes. Fuzzy models are easily interpretable and very powerful when it comes to approximating arbitrary nonlinear functions. Because of its simplicity and the use of linear programming, fuzzy linear regression is frequently used to evaluate the relationship between the dependent and independent variables of a fuzzy model [17].

In seeking a means of describing the domain of functions that result from an uncertain system, one way to define a fuzzy model is to fix certain membership functions and establish the consequence parameters that vary in a certain interval. By applying only one model with interval parameters, one is able to define the upper and lower boundaries of a given band. As was introduced and shown in [18], the optimization of the interval fuzzy model (INFUMO) parameters based on partial fuzzy linear programming is easy to implement and is not computationally demanding.

When process parameters vary in a certain tolerance band, it is advantageous to define a confidence band over a finite set of input and output measurements in which the effects of unknown process inputs are already included. The main idea of the proposed approach is to apply the INFUMO to provide the boundary functions of a given parameter interval and use it in a fault-detection system as a residual generator. By calculating the normalized distance of the process output from the boundary model outputs, a numerical fault measure is obtained.

The problem of designing a robust fault-detection system has been extensively studied over the past two decades. The main challenge is to construct a residual generator that will be insensitive to the influences of disturbances and model uncertainties. However, if the uncertainties are unstructured only an approximate decoupling among the faults and the uncertainties can be reached. In [4] a generalized residual scheme based on a factorization technique and the use of a postfilter has been proposed. A large number of robust postfilter design algorithms have been developed in recent years: the eigenvalue-eigenvector algorithm in the time domain [10], frequency-domain-based algorithms using H_2 [8] and H_∞ theory [14], [11], [20], algorithms for nonlinear processes [15], and the use of nonlinear adaptive observers [3], [5]. All of the above-mentioned methods have something in common: that the transfer functions from faults and unknown inputs to the output must be known in order to draw a distinction between a fault and the influence of unknown inputs. For a class of interval-type uncertain systems where the influence of unknown inputs cannot be modelled, the proposed approach is more appropriate because both the influences of unknown inputs and parameter change are included in the model.

The objective of this work is to present a case where by using the proposed method simple solutions for detecting load change in a motor-generator plant can be developed. One of the major advantages of INFUMO identification is that arbitrary excitation signals can be used. Optimization convergence problems that might arise either from too many parameters or from a vast amount of data were solved by implementing a simple data-reduction method, and low-pass filtering [13].

The paper is organized in the following way. In Section 2 the preliminaries of interval arithmetics, the background of fuzzy modelling, the main idea of interval fuzzy model identification using l_∞-norm, and the residual formation with a fault-diagnostic scenario are described. Section 3 presents the motor-generator plant that was used as a system with uncertain parameters, and the application to load-change detection with data pre-processing and low-pass filtering is introduced. In the final part, some outlines of future work are given.

II. Using the fuzzy interval model in fault detection

In this section the INFUMO will be introduced in a fault-detection system for a class of processes with interval-type parameters.

A. Preliminaries

A class of systems containing parameters described by a set of bounded variables will be considered. A system from this class has an output described by a domain, which includes all possible normal behaviours. Interval arithmetic is used to describe the whole continuous range of behaviours represented by an interval model. Let x be a real value that can be found in the bounded space $\mathcal{S}(x) = [\underline{x}, \overline{x}] = \{x \in \mathbb{R}, \underline{x} \leq x \leq \overline{x}\}$. If $f(x)$ is a continuous and differentiable function, $\mathcal{S}(f(x))$ is a natural extension of the function [1], [7] described as

$$\mathcal{S}\left(f(x)\right) = \left[\inf_{x\in\mathcal{F}(x)}(f(x)) \quad \sup_{x\in\mathcal{F}(x)}(f(x))\right] \tag{1}$$

Let the process output be given as follows:

$$y(t) = G(p)u(t) + G_D(p)d(t) + n(t) \tag{2}$$

where p denotes the forward shift operator, $G(p)$ the process transfer function, $d(t)$ is a vector of disturbances, $G_D(p)$ the unknown disturbance transfer function, and $n(t) \in [\underline{n}, \overline{n}]$ denotes a bounded parameter from the space $\mathcal{S}$. Using Eq. (1), the boundary output functions can be described as

$$\begin{aligned} \overline{y}(t) &= f(u,d,t,\overline{n}) \in \mathcal{S}\big(y(n(t))\big) \\ \underline{y}(t) &= f(u,d,t,\underline{n}) \in \mathcal{S}\big(y(n(t))\big) \end{aligned} \tag{3}$$

As a consequence, a confidence band of outputs guarantees that a process output exhibiting normal behaviour is found in the interval $\left[\underline{y}, \overline{y}\right]$. However, due to the unknown effect of disturbances the exact bounds cannot be defined analytically. In [7] two approaches to determine the parameter-uncertainty values have been proposed: empirical and numerical. In the first approach, physical knowledge of the uncertainties is used to adjust its values in the model. The second approach consists of using a constrained linear optimization technique to minimize the model precision objective function $J = 1/N\sum_{k=1}^{N}(\overline{y}_i - \underline{y}_i)$. The proposed approach, introducing the interval fuzzy model, is qualitatively different because the boundary responses will be obtained by a fuzzy function approximation of the bounds of a set of input-output data that already comprises the effect of disturbances.

B. Derivation of an interval fuzzy model

The derivation of an interval fuzzy model can be roughly divided into the following stages: applying a fuzzy model in Takagi-Sugeno form [16], interval identification using l_∞-norm and obtaining an interval fuzzy model using partial linear programming. A short description of all the stages will be given next. A more detailed insight can be found in [18].

A static fuzzy TS-type model in affine form can be given as a set of rules

$$\mathbf{R}_\mathrm{j}: \ \mathbf{if}\, \mathrm{x_p} \ = \ \mathbf{A}_\mathrm{j}, \ \mathbf{then}\, \mathrm{y} = \boldsymbol{\theta}_\mathrm{j}^\mathrm{T}\mathbf{x}, \ \mathrm{j} = 1,\ldots,\mathrm{m} \tag{4}$$

The variable x_p denotes the input or variable in premise, and variable y is the output of the model. The antecedent variable is connected with m fuzzy sets $\mathbf{A}_j$, and each fuzzy set $\mathbf{A}_j$ $(j = 1,\ldots,m)$ is associated with a real-valued function $\mu_{A_j}(x_p) : \mathbb{R} \to [0,1]$, that produces a membership grade of the variable x_p with respect to the fuzzy set $\mathbf{A}_j$. The consequent vector is denoted $\mathbf{x}^T = [x, 1]$. As the output functions are in affine form, 1 was added to the vector $\mathbf{x}$. The system output is a linear combination of the consequent states, and $\boldsymbol{\theta}_j$ is a vector of fuzzy parameters.

If the intersection of the fuzzy sets is previously defined, the system in Eq. (4) can be described in closed form

$$y = \boldsymbol{\beta}^T(x_p)\boldsymbol{\Theta}\mathbf{x}, \tag{5}$$

where $\boldsymbol{\Theta}^T = [\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_m]$ denotes a coefficient matrix for the complete set of rules, and $\boldsymbol{\beta}^T(x_p) = [\beta_1(x_p), \ldots, \beta_m(x_p)]$ is a vector of normalized membership functions with elements that indicate the degree of fulfilment of the respective rule. Functions $\beta_j(x_p)$ can be defined as

$$\beta_j(x_p) = \frac{\mu_{A_j}(x_p)}{\sum_{j=1}^{m}\mu_{A_j}(x_p)}, \ j = 1,\ldots,m, \tag{6}$$

if the intersection of the fuzzy sets is defined as the *triangular norm* (T-norm) and if the partition of unity is assumed. In our case a simple algebraic product was chosen as the T-norm.

A model parameter estimation using l_∞-norm as a criterion for the measure of the modelling error will be considered next. Let $\mathbf{C} \subset \mathbb{R}$ be a compact set and $\mathcal{G} = \{g : \mathbf{C} \to \mathbb{R}\}$ be a class of nonlinear functions. Let us assume that there exist the exact upper bound $\overline{g}$ and the exact lower bound $\underline{g}$ that satisfy the following conditions for each z and an arbitrary $\varepsilon > 0$:

$$\overline{g}(z) \geq \max_{g\in\mathcal{G}} g(z), \ \exists g \in \mathcal{G} : \overline{g}(z) < g(z) + \varepsilon \tag{7}$$

$$\underline{g}(z) \leq \max_{g\in\mathcal{G}} g(z), \ \exists g \in \mathcal{G} : \underline{g}(z) > g(z) + \varepsilon \tag{8}$$

Obtaining the bounds in Eqs. (7) and (8) would require an infinite amount of data; however, in this case we are limited to the finite set of measured output values $\mathbf{Y} = \{y_1, y_2, \ldots, y_N\}$ and the finite set of input data $\mathbf{Z} = \{z_1, z_2, \ldots, z_N\}$:

$$y_i = g(z_i), \quad g \in \mathcal{G}, z \in \mathbf{C} \subset \mathbb{R}, y_i \in \mathbb{R}, i = 1,\ldots,N \tag{9}$$

Therefore, the upper and the lower boundary functions are approximated by fuzzy functions in the form given in Eq. (5). According to the Stone-Weierstrass Theorem [12], [19], there exists a fuzzy system f such that

$$\max_{z_i\in\mathbf{Z}}|f(z_i) - g(z_i)| < \varepsilon, \quad \forall i, \tag{10}$$

i.e., a fuzzy function can approximate an arbitrary function $g \in \mathcal{G}$ with any desired degree of accuracy for any ε.

To estimate the optimal parameters of the proposed fuzzy function the minimization of the maximum modelling error

$$\max_{z_i\in\mathbf{Z}}|y_i - f(z_i)| = \max_{z_i\in\mathbf{Z}}\left|y_i - \boldsymbol{\beta}^T(x_p)\boldsymbol{\Theta}\mathbf{x}(z_i)\right| \tag{11}$$

over the whole input set $\mathbf{Z}$ is performed. This implies the *min-max* optimization method, and l_∞-norm is used as the modelling error measure. Note that the data are obtained by sampling different functions from $\mathcal{G}$ with arbitrary values of z.

The idea of robust fuzzy interval modelling can be seen as finding a lower fuzzy function $\underline{f}$ and an upper fuzzy function $\overline{f}$ that satisfy the following condition:

$$\underline{f}(z_i) \leq y_i \leq \overline{f}(z_i), \quad \forall z_i \in \mathbf{Z}. \tag{12}$$

The main requirement when defining the band is that it is as narrow as possible, within the proposed constraints.

The upper and the lower fuzzy functions, respectively, can be found by solving the following optimization problems for $\forall i$:

$$\begin{aligned}
&\min_{\underline{\boldsymbol{\Theta}}} \max_{z_i \in \mathbf{Z}} \left| y_i - \boldsymbol{\beta}^T(\mathbf{x}_{pi})\underline{\boldsymbol{\Theta}}\mathbf{x}(z_i) \right|, \text{ if } y_i - \boldsymbol{\beta}^T(\mathbf{x}_{pi})\underline{\boldsymbol{\Theta}}\mathbf{x}(z_i) \geq 0,\\
&\min_{\overline{\boldsymbol{\Theta}}} \max_{z_i \in \mathbf{Z}} \left| y_i - \boldsymbol{\beta}^T(\mathbf{x}_{pi})\overline{\boldsymbol{\Theta}}\mathbf{x}(z_i) \right|, \text{ if } y_i - \boldsymbol{\beta}^T(\mathbf{x}_{pi})\overline{\boldsymbol{\Theta}}\mathbf{x}(z_i) \leq 0.
\end{aligned} \tag{13}$$

The solutions to both problems can be found by fuzzy linear programming, because both problems can be viewed as linear programming problems. This brings simplicity to the realization of the optimizing process. However, large data sets and a large number of parameters will still pose a threat to optimization convergence. In the first case we approach the problem with data-reduction methods, and in the latter case, on the other hand, we have to find solutions to reduce the number of parameters.

C. Residual formation and diagnostic scenario

As was shown in [6], [9], all residual generators can be designed by

$$r(t) = Q(p)\big(y(t) - \tilde{y}(t)\big), \tag{14}$$

with $\tilde{y}(t)$ as an output estimation and $Q(p)$ is a filter which is free to design and enhances the residual robustness to unknown process inputs. Combining Eq. (14) with Eq. (13), the following relation can be written:

$$\begin{aligned}
r(t) &= Q(p)\big(y(t) - \boldsymbol{\beta}^T\boldsymbol{\Theta}\mathbf{u(t)}\big)\\
&= Q(p)y(t) - Q(p)\boldsymbol{\beta}^T\boldsymbol{\Theta}\mathbf{u(t)}\\
&= y_f(t) - \boldsymbol{\beta}^T\boldsymbol{\Theta}\mathbf{u_f(t)}
\end{aligned} \tag{15}$$

where $\mathbf{u}(t) = [u(t)\ 1]^T$ denotes the augmented input vector. The main idea of the proposed approach is to filter both the input and the output data, thus obtaining a confidence band of filtered input-output data pairs, approximate the band using the optimization procedure of the INFUMO, and connect the INFUMO in parallel to the process to get online estimations of the boundary outputs. For fault detection, the decision function should consist of verifying that each measurement belongs to the corresponding confidence band. In order to provide quantitative information about the proximity of the measurements to the closest interval bound, distances were used, as presented in [7].

If a filtered output value $y_f(t)$ belongs to an interval $\left[\underline{y}_f(t), \overline{y}_f(t)\right]$, and if the mean interval value is denoted $\hat{y}_f(t)$, the proposed distance is defined in the following way:

$$\begin{aligned}
&\text{if } y_f(t) < \hat{y}_f(t),\ d(y_f) = \frac{y_f(t) - \hat{y}_f(t)}{\underline{y}_f(t) - \hat{y}_f(t)}\\
&\text{if } y_f(t) > \hat{y}_f(t),\ d(y_f) = \frac{y_f(t) - \hat{y}_f(t)}{\overline{y}_f(t) - \hat{y}_f(t)}
\end{aligned} \tag{16}$$

The distance in (16) is zero when the measurement is equal to $\hat{y}_f$, and approaches the value 1 if the measurement is close to one of the interval bounds. A fault is signalled every time $d(y_f)$ exceeds the value 1. Fig. 1 gives a schematic representation of the proposed fault-detection system. The filter $Q(p)$ is represented by a block denoted LPF, and the distance is calculated in the DIST block.

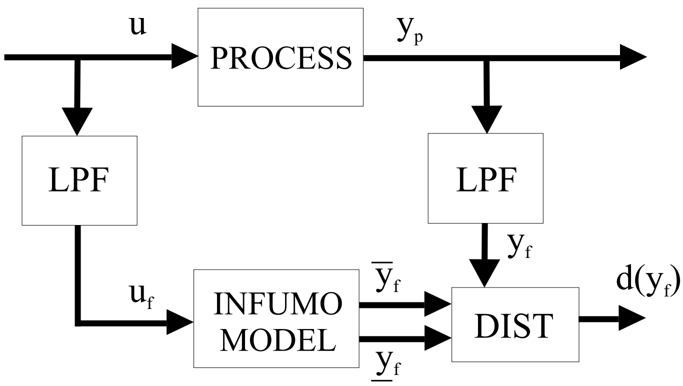

Fig. 1. Fault-detection system using static INFUMO model

III. Application of Infumo in the fault detection of a motor-generator plant

In this section the application of the INFUMO in the robust identification and fault detection (FD) of a process from a class of systems with uncertain and interval-type parameters will be presented.

The electromechanical process consists of 2 DC motors, mounted facing each other, as shown in Fig. 2. The driving shafts are rigidly coupled. The left motor, marked as 'G', is the load of the motor 'M' when operating in generator mode. Applying a negative voltage to the generator produces mechanical torque and results in a shift of the operating conditions. The system output is the voltage obtained by a tacho generator, mounted to the shaft, that converts the rotary speed to a DC-voltage output signal. u_m and u_g are the input voltages for the excitation and the load, respectively. The signals are connected through an AD/DA converter to a PC. The plant setup enables one to control the shaft speed by changing the motor's input voltage.

The process parameters are uncertain. If consecutive open-loop experiments on identical input signals are performed, the output responses will form a set of different trajectories rather than a single one. One of the reasons for such behavior is that the system performance depends on the operating temperature.

Experiments show that load values ranging from $u_g = 0$ V to $u_g = -0.05$ V do not shift the operating conditions

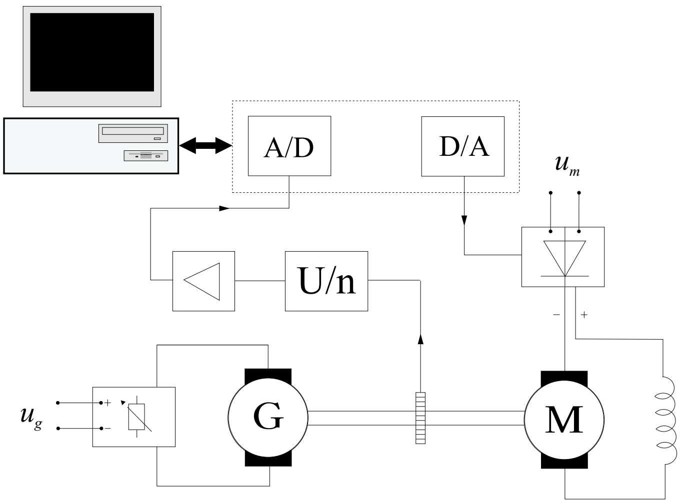

Fig. 2. Schematic representation of the motor-generator plant

substantially. Hence, the confidence load interval was defined as $[-0.05, 0]$ V. With reference to the given INFUMO identification procedure, a confidence band of input-output data must be defined. This band will represent the most significant operating range of the plant and also include all unexpected deviations due to parameter uncertainties. A set of 30 experiments was carried out, i.e., 5 series of 6 identification signals at load voltages from the lowest to the highest value in 0.01 V steps. The inputs and associated output signals are shown in Fig. 3. For the sake of brevity, only the first, the second, and the last data sets are presented.

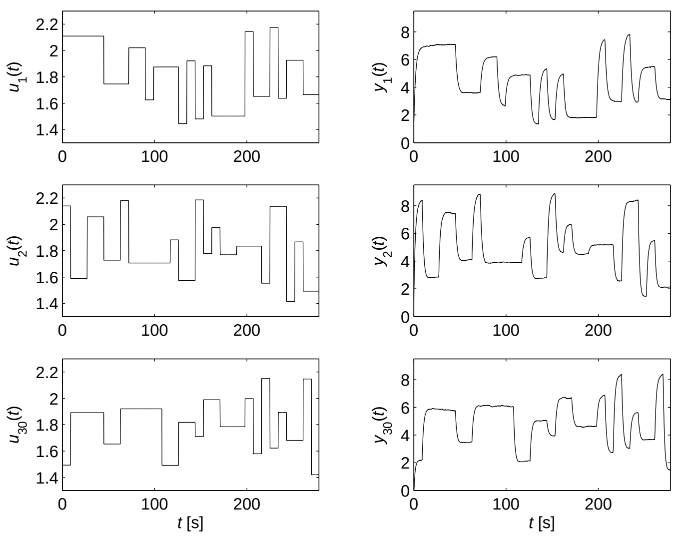

Fig. 3. Inputs and outputs: the first, the second, and the last experiment

One of the major benefits of the interval fuzzy model identification, shown in Fig. 3, is that the input signals can be arbitrary. Normally, to get a confidence band of measurements, it would be necessaray for the experiments to be conducted with identical excitation signals. Our idea, however, was to create a model based on data acquired from experiments on unequal signals.

According to Eq. 14, the input and output signals are subjected to low-pass filtering. The structure of the LPF was chosen as a simple first-order system, represented by the transfer function in Eq. (17)

$$G_f = \frac{1}{T_f s + 1} \tag{17}$$

Optimal design of the LPF time constant was not considered in this study. The cut-off frequency must be chosen to be low enough to let only slowly changing signals propagate through the filter. As this directly affects the choice of the time constant, a compromise has to be made in order that the system response is not too slow. Hence, it was chosen as $T_f = 30$ s. This way a compact set of measurements that represents steady-state system behaviour is obtained. It can be seen as a load-dependent static input-output mapping area.

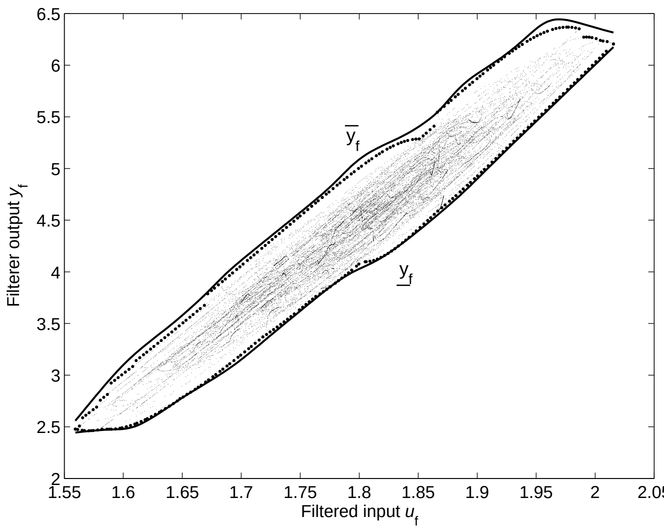

Fig. 4. Set of filtered input-output data with boundary points and boundary INFUMO functions

The total number of points gathered from the identification experiments was 83700. Performing the optimization on the given data set would be extremely time consuming. Therefore, data reduction is performed by determining the boundary points. Firstly, the range of input measurements is divided into equidistant subspaces. The length of the step is chosen according to the subspace with the highest density of data. In each subspace the extremal points are determined. The input-output data is presented in Fig. 4, and the resulting set of 302 boundary points is emphasized. These data will be used as the training data set for the INFUMO identification. A static INFUMO can be employed. This brings an additional reduction of fuzzy parameters to be optimized. The membership functions of the INFUMO antecedent variables were arranged using the Gustafson-Kessel clustering method [2]. According to the shape of the data area, it was sufficient to use 6 fuzzy subsets for the upper and lower fuzzy functions.

The parameters were optimized using the proposed INFUMO optimization algorithm in Eq. 13. The resulting boundary functions can be seen in Fig. 4. It is evident that the *min-max* optimization gave satisfactory results in approximating the given area.

To realize a fault-detection system, INFUMO is connected to the process in parallel, as shown in Fig. 1. In the test experiment the load signal was a combination of ramps that is outside the load band in the time period $T_{ft} = 160 - 330$ s. It is presented in Fig. 5 along with the input test signal and the corresponding process output signal. In the first 140 seconds the input signal was constant, so the operating conditions were met. The results of the test run can be seen from Fig. 6. It is evident that the proposed FD system successfully tracks the load crossing of the permitted band. The FD output is higher than the one in the shaded area, denoted 'Load boundary crossing',

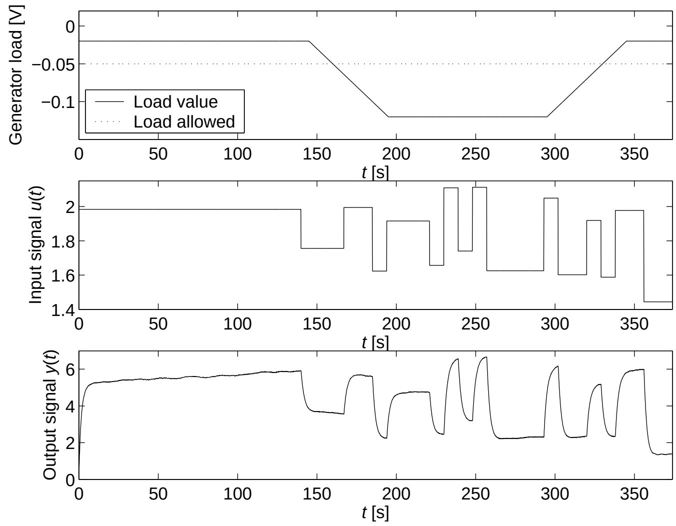

Fig. 5. Test experiment signals

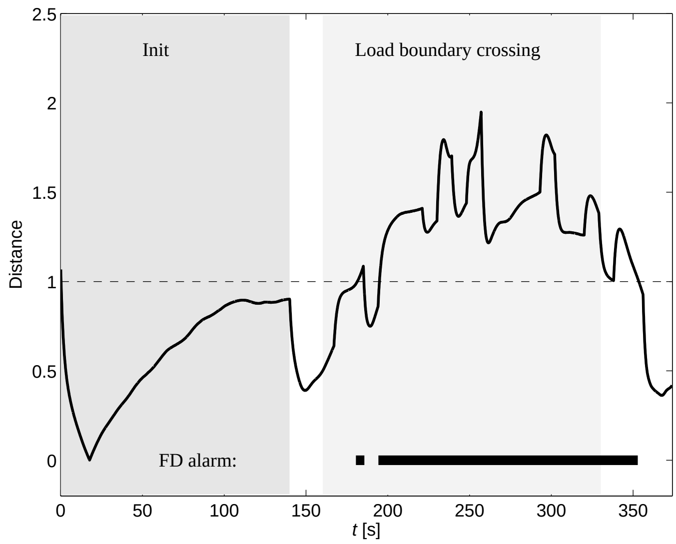

Fig. 6. Results of the fault detection system

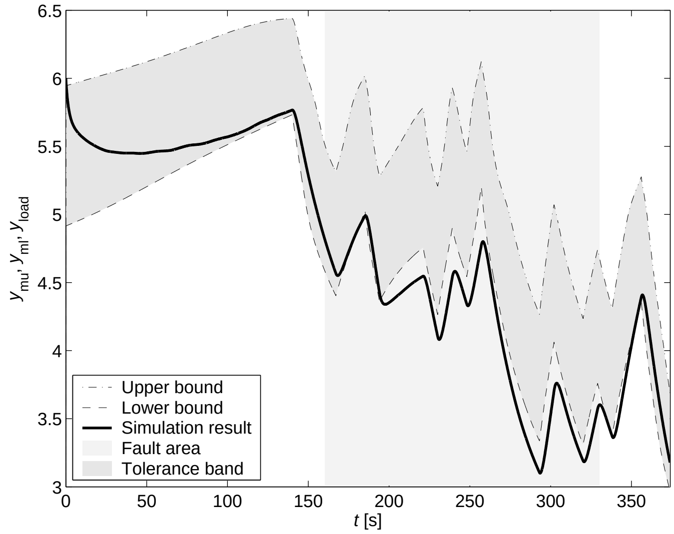

Fig. 7. Results of simulation with variable load

with a time delay, depending on the time constant of the proposed low-pass filter. On one occasion the alarm was not called correspondingly to the load value; this will be discussed with reference to Fig. 7, which displays time-dependent courses of the filtered process output and the INFUMO tolerance band. It can be seen that during the transient the filtered process output crossed the boundary area for a short period of time. It can be concluded that in this case fault prediction was not certain due to the effect of plant unmodelled dynamics. One way of reducing this uncertainty would be to find optimal structure and parameters of the applied filter, but that was not considered in this work.

Fig. 8 shows the actual view of fault detection from the INFUMO static model perspective. It can be clearly seen

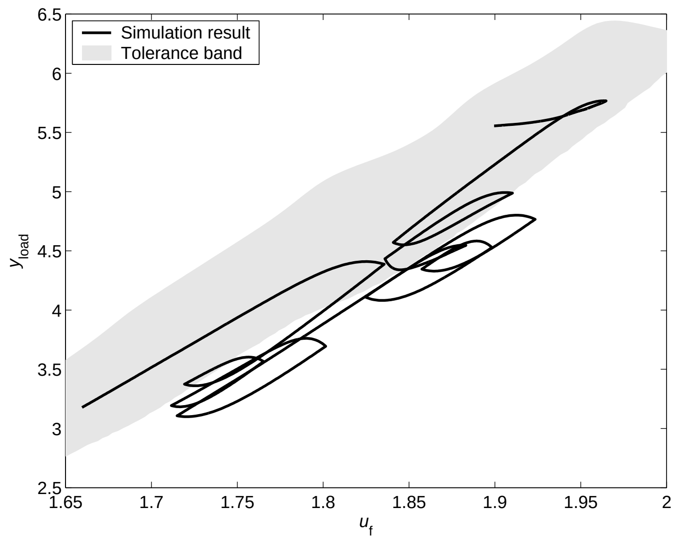

Fig. 8. Results of fault detection system in static conditions

in Fig. 8 that when the load value is below the negative limit, the filtered response abandons the tolerance band.

In addition, the above-mentioned case of false-error estimation can be clearly noted.

IV. Conclusion

A novel approach of interval fuzzy model identification has been applied in fault detection. The interval fuzzy model (INFUMO) was derived using the l_∞-norm function approximation. It was shown that the INFUMO enables the confining of an arbitrary nonlinear confidence band with an upper and lower fuzzy function. It is therefore suitable for the identification of systems with uncertain parameters, as all the system responses in the given interval of uncertainty can be found in the confidence band with a certainty of 1. The benefit in fault detection is to be able to directly model a family of interval-type parameter systems, which guarantees fault-tolerant action.

An application involving the load-change detection of a motor-generator pilot plant was presented. To get a confidence band of system responses, a large number of experiments was carried out, which resulted in a huge set of data. The problem of data reduction was dealt with by filtering the input and output signals using a low-pass filter. The resulting major benefits were the possibility of using arbitrary input signals and the simplicity of the fuzzy static model that was used. The boundary points of the gathered data set were determined using a simple algorithm and used as a training data set for identification by linear programming. Connecting the INFUMO to the process in parallel and employing an online calculation of the normalized distance of the filtered process output from the nearest bound, the proposed approach was proven to be successful in detecting unwanted load changes.

Future work will concentrate on an optimization of the filter parameters, investigating the performance resulting from different choices of filter structure, and investigating possible extensions to frequency-based methods and fault-tolerant control.

References

[1] J. Armengol, L. Travé-Massuyès, J. Vehí, and J. L. de la Rosa, "A survey on interval model simulators and their properties related to fault detection," in *Proceedings of the 14th IFAC world congress*, vol. O, 1999, pp. 511–519.

[2] R. Babuška, *Fuzzy Modeling for Control.* Boston, USA: Kluwer Academic Publishers, 1998.

[3] G. Bastin and M. R. Gevers, "Stable adaptive observers for nonlinear time-varying systems," *IEEE Trans. Contr.*, vol. AC, no. 33, pp. 650–658, 1988.

[4] X. Ding and P. M. Frank, "Fault detection via factorization approach," *Syst. Contr. Lett.*, vol. 14, pp. 431–436, 1990.

[5] ——, "On-line fault detection in uncertain systems using adaptive observers," *European J. of Diagnosis and Safety in Automation*, vol. 3, pp. 9–21, 1993.

[6] X. Ding, L. Guo, and P. M. Frank, "Parameterization of linear observers and its application to observer design," *IEEE Transactions on Automatic Control*, vol. 39, no. 8, pp. 1648–1652, 1994.

[7] I. Fagarasan, S. Ploix, and S. Gentil, "Causal fault detection and isolation based on a set-membership approach," *Automatica*, vol. 40, pp. 2099–2110, 2004.

[8] P. M. Frank, "Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy - a survey," *Automatica*, vol. 26, pp. 459–474, 1990.

[9] P. M. Frank and X. Ding, "Survey of robust residual generation and evaluation methods in observer-based fault detection systems," *J. Proc. Control*, vol. 7, no. 6, pp. 403–424, 1997.

[10] P. M. Frank and J. Wünnenberg, *Fault Diagnosis in Dynamic Systems.* Prentice Hall, 1989, ch. Robust fault diagnosis using unknown input observer schemes, pp. 46–98.

[11] E. Frisk and L. Nielsen, "Robust residual generation for diagnosis including a reference model for residual behavior," in *Proceedings of the 14th IFAC world congress*, vol. P, Beijing, China, 1999, pp. 55–60.

[12] R. R. Goldberg, *Methods of Real Analysis.* John Wiley and Sons, 1976.

[13] R. Isermann, K. H. Lachmann, and D. Matko, *Adaptive Control Systems.* New York: Prentice Hall, 1992.

[14] Z. Qiu and J. Gertler, "Robust FDI systems and H_∞-optimization," in *Proceedings of the 32nd CDC*, San Antonio, USA, 1993.

[15] R. Seliger and P. M. Frank, "Fault diagnosis by disturbance decoupled nonlinear observers," in *Proceedings of the 30th IEEE CDC*, vol. 3, 1991.

[16] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modelling and control," *IEEE Trans. on Systems, Man, and Cybernetics*, vol. 15, pp. 116–132, 1985.

[17] H. Tanaka, S. Uejima, and K. Asai, "Linear regression anaysis with fuzzy model," *IEEE Transactions on Systems, Man and Cybernetics*, vol. SMC-12, no. 6, pp. 903–907, 1982.

[18] I. Škrjanc, S. Blažič, and O. Agamennoni, "Identification of dynamical systems with a robust interval model," *Automatica*, vol. 41, no. 2, 2005, in press.

[19] H. Ying and G. Chen, "Necessary conditions for some typical fuzzy systems as universal approximators," *Automatica*, vol. 33, pp. 1333–1338, 1997.

[20] M. Zhong, S. X. Ding, J. Lam, and H. Wang, "An LMI approach to design robust fault detection filter for uncertain LTI systems," *Automatica*, vol. 39, pp. 543–550, 2003.