Binary Decoding of Concatenated Turbo Codes and Space-Time Block Codes for Quaternary Modulations

***Abstract*— Space-time block (STB) coding is one of the key techniques used to combat fading in multi-antenna wireless communications systems. STB coding provides diversity gain while having a simple decoding. However, when further gains are needed, an outer channel code has to be concatenated. Turbo codes are channel codes that have been shown to perform close to the Shannon limits in AWGN channels. Moreover, they have a relatively low-complexity binary MAP decoding. However, this complexity increases exponentially with the number of bits per symbol. In this paper, a system consisting on an outer turbo code concatenated with a STB code is considered. A general formulation for STB decoding based on a least squares criteria is given in order to allow binary turbo decoding while using quaternary modulations such as QPSK. This binary decoding results in a higher global rate at no complexity cost. A normalization algorithm of STB codes is also given in order to present a fair comparison in terms of energy between MIMO (multiple-input multiple-output) and single-antenna systems.**

I. Introduction

Future wireless communications systems are dealing with the need to provide high-rate data communications over time-varying fading channels. In order to provide robust communications in this environment, it is common to use a channel code. Turbo codes are channel codes that have been shown to yield remarkable coding gains close to Shannon limits in AWGN channels [1]. However, they were thought for AWGN channels and they suffer a loss in performance when they have to face fading channels. In this sense, if further gains are needed, some kind of diversity has to be implemented.

Space-Time Turbo Codes (STTC) [2-3] have been proposed as an alternative that integrates space-time and turbo coding into one single structure. However, they have severe complexity drawbacks at the receiver. In addition, the global design hardly depends on the antennas configuration. It this sense, a change in the number of transmit and/or receive antennas forces a change in the structure of both the turbo coder and decoder, dramatically reducing the system flexibility.

Space-Time Block (STB) coding [4-5] is a combination of spatial and temporal diversity at the transmitter that provides important diversity gains while using linear processing at the receiver. It is also interesting in the sense that places all the intelligence at the transmitter while leaving low complexity at the receiver. Thinking in a cellular system, the cost of multiple transmit chains at the base stations can be amortized over the total number of users.

This work considers the serial concatenation of a turbo code and a STB code [6-8]. By separating the channel coding from the diversity coding, the system presents a modular structure whose key advantages are flexibility and low complexity. Then, the iterative decoding of turbo codes is reasonably simple. This reasonable complexity applies to the case of binary modulations, as the complexity of turbo decoding increases exponentially with the number of bits per symbol. However, the use of binary modulations such as BPSK results in a poor global rate. The main issue addressed in this work is to increase the data rate by extending the binary decoding to quaternary modulations such as QPSK with no increase in turbo decoding complexity. This is achieved by STB decoding and estimating at bit level, splitting the quaternary QPSK modulation into two binary BPSK channels.

The outline of the paper is the following. Section II gives a brief system description and introduces the signal model. Formulation and least squares decoding of STB codes is addressed in Section III. Section IV is concerned with the binary turbo decoding of quaternary modulations. Performance evaluation of the system under two different configurations is given in Section V. Finally, some conclusions are summarized in Section VI.

II. System Description and Signal Model

The global system structure is shown in Figure 1.

A frame of N_{TC} data bits is turbo encoded (optionally punctured) and BPSK modulated. The structure of the turbo code is based on the parallel concatenation of two Recursive Systematic Convolutional (RSC) codes [1]. One RSC code codes the N_{TC} data bits while the other codes and interleaved version of the data bits. The data bits and the parity branch of each RSC code is then multiplexed to form a codeword.

Then, the encoded BPSK symbols are QPSK mapped in couples and grouped into blocks of Q QPSK symbols. The in-phase and quadrature components are denoted by $\underline{\alpha}(n)$ and $\underline{\beta}(n)$ respectively, and the index n stands for the position of the block over the total number to be STB coded in the same frame. Each of these blocks is then STB coded along T channel uses and M transmit antennas.

The samples taken over the N receive antennas along T time slots (channel uses) allow the STB decoder to estimate the in-phase and quadrature components, $\underline{\hat{\alpha}}(n)$ and $\underline{\hat{\beta}}(n)$, for the Q symbols of the current STB block. Then, they are serially grouped until the estimates of all the blocks relative to the same frame have been obtained. Finally, the turbo decoder can start the decoding of the symbols.

From the turbo coder and decoder point of view, the encoded BPSK symbols see a SISO (single-input single-output) equivalent channel (as shown in the figure, dashed line). In consequence, a binary decoding can be done while

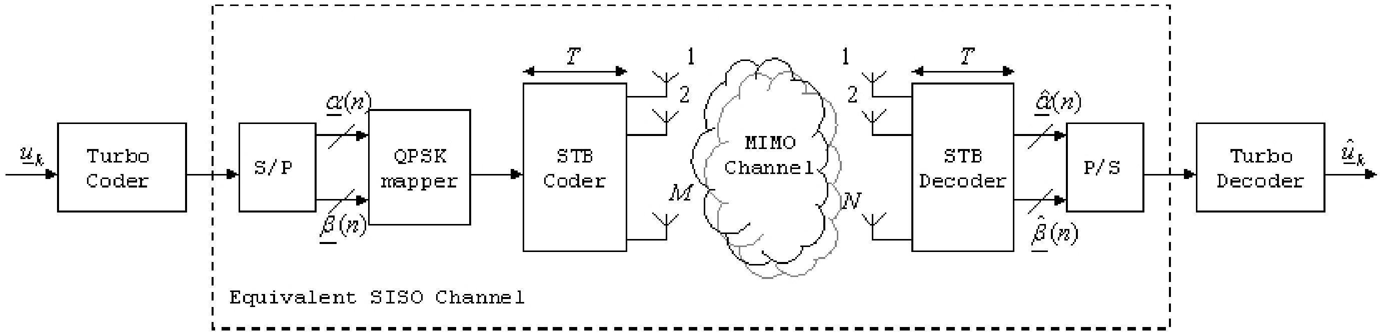

Fig. 1. Block diagram of the system

using a quaternary modulation such as QPSK, keeping a low-complexity receiver while avoiding low-rate modulations.

III. Formulation and Decoding of STB Codes

The main aim of this section is to find the relationship between the in-phase and quadrature components of the transmitted symbols, $\underline{\alpha}(n)$ and $\underline{\beta}(n)$, and the received samples during a block period (T time slots).

A. General Formulation of STB codes

A STB code takes a block of Q symbols as inputs and transmits linear combinations of them and their complex conjugates over M antennas in T channel uses. The rate of the code is then Q/T. Using the following matrix formulation, it can be described as:

$$\mathbf{S}(n) = \sum_{q=1}^{Q} \alpha_q(n)\mathbf{A}_q + j\beta_q(n)\mathbf{B}_q \tag{1}$$

where $\mathbf{S}(n)$ is a matrix containing T column vectors, each one representing the transmitted symbols over the M antennas at a given channel use, that is:

$$\mathbf{S}(n) = [\underline{s}_1(n) \quad \underline{s}_2(n) \quad ... \quad \underline{s}_T(n)] \tag{2}$$

$$\underline{s}_i(n) \in \mathbb{C}^{M\times 1} \quad \text{for} \quad i = 1...T$$

$\{\mathbf{A}_q\}_{q=1...Q}$, $\{\mathbf{B}_q\}_{q=1...Q}$ and $\mathbf{S}(n)$ are $M \times T$ complex matrices, that is, $\{\mathbf{A}_q, \mathbf{B}_q, \mathbf{S}(n)\} \in \mathbb{C}^{M\times T}$. The set of matrices $\{\mathbf{A}_q\}$, $\{\mathbf{B}_q\}$ represents the contribution of the real ($\alpha_q(n)$) and imaginary part ($\beta_q(n)$) of the $q-th$ symbol to the $n-th$ transmitted block $\mathbf{S}(n)$.

It is clear that the use of a STB code increases the effective energy per symbol if no normalization is done, since the energy of Q symbols is potentially transmitted along T channel uses. In order to obtain coherent simulation results in a fair comparison with SISO systems, this issue has to be taken into account. A normalization algorithm for the given set of constituent matrices $\{\mathbf{A}_q\}_{q=1...Q}$, $\{\mathbf{B}_q\}_{q=1...Q}$ is explained in Appendix I.

B. Matrix Model of STB Reception

A model in matrix notation is derived from [9] for the reception of STB codes. Its application to obtain the symbols' estimates will be considered herein.

We now assume that the normalization of the STB codes has been done, which only represents some row-by-row scaling factors in the set of matrices $\{\mathbf{A}_q\}$, $\{\mathbf{B}_q\}$. Then, we focus in the received block $\mathbf{Y}(n)$, that can be expressed as

$$\mathbf{Y}(n) = \mathbf{HS}(n) + \mathbf{W}(n) \tag{3}$$

where $\mathbf{H} \in \mathbb{C}^{N\times M}$ stands for the MIMO channel matrix, whose components are zero mean complex gaussian with unit variance, and $\mathbf{W}(n) \in \mathbb{C}^{N\times M}$ is the matrix whose entries correspond to thw AWGN samples of the $n-th$ space-time block.

It is interesting for the subsequent analysis to obtain a closed expression for the vertical stacking of $\mathbf{S}(n)$ denoted by *vec*$(\mathbf{S}(n))$.

$$vec(\mathbf{S}(n)) = \sum_{q=1}^{Q} \alpha_q(n) vec(\mathbf{A}_q) + j\beta_q(n) vec(\mathbf{B}_q) \tag{4}$$

Using the following definitions:

$$\underline{a}_q \equiv vec(\mathbf{A}_q) \quad q = 1...Q$$
$$\underline{b}_q \equiv vec(\mathbf{B}_q) \quad q = 1...Q$$
$$\tilde{\mathbf{A}} \equiv [\underline{a}_1(n) \quad \underline{a}_2(n) \quad ... \quad \underline{a}_Q(n)]$$
$$\tilde{\mathbf{B}} \equiv [\underline{b}_1(n) \quad \underline{b}_2(n) \quad ... \quad \underline{b}_Q(n)]$$

expression (4) can be rewritten as

$$vec(\mathbf{S}(n)) = [\tilde{\mathbf{A}} \quad j\tilde{\mathbf{B}}] \begin{bmatrix} \underline{\alpha}(n) \\ \underline{\beta}(n) \end{bmatrix} \equiv [\tilde{\mathbf{A}} \quad j\tilde{\mathbf{B}}]\underline{x}(n) \tag{5}$$

Then, by vertically stacking the received block of samples, we have

$$\underline{y}(n) \equiv vec(\mathbf{Y}(n)) = vec(\mathbf{HS}(n)) + vec(\mathbf{W}(n)) \tag{6}$$

In order to obtain the estimates of the in-phase and quadrature components of the symbols, it is necessary to separate all the matrices into its real and imaginary parts.

$$\mathbf{H} = \mathbf{H}_R + j\mathbf{H}_I \tag{7}$$

$$\underline{w}(n) \equiv \mathit{vec}(\mathbf{W}(n)) = \underline{w}_R(n) + j\underline{w}_I(n) \tag{8}$$

The following property attaining to the vertical stacking of the product of matrices [10] will also be used:

$$\mathit{vec}(\mathbf{LRC}) = (\mathbf{C}^T \otimes \mathbf{L})\mathit{vec}(\mathbf{R}) \tag{9}$$

where $(.)^T$ stands for matrix transposition and $\otimes$ for the Kronecker's product. Then, the use of (9) into (6) yields:

$$\underline{y}(n) = (\mathbf{I}_T \otimes \mathbf{H})\mathit{vec}(\mathbf{S}(n)) + \underline{w}(n) \tag{10}$$

where $\mathbf{I}_T$ stands for the $T \times T$ identity matrix. Now, we can apply (5),(7) and (8) into (10), and after some manipulation and separation of the real and imaginary parts of $\underline{y}(n)$ we can obtain:

$$\underline{\tilde{y}}(n) \equiv \left[\begin{array}{c} \underline{y}_R(n) \\ \underline{y}_I(n) \end{array} \right] =$$

$$= \underbrace{\left[\begin{array}{cc} (\mathbf{I}_T \otimes \mathbf{H}_R)\tilde{\mathbf{A}} & -(\mathbf{I}_T \otimes \mathbf{H}_I)\tilde{\mathbf{B}} \\ (\mathbf{I}_T \otimes \mathbf{H}_I)\tilde{\mathbf{A}} & (\mathbf{I}_T \otimes \mathbf{H}_R)\tilde{\mathbf{B}} \end{array} \right]}_{\mathbf{F}} \left[\begin{array}{c} \underline{\alpha}(n) \\ \underline{\beta}(n) \end{array} \right] + \left[\begin{array}{c} \underline{w}_R \\ \underline{w}_I \end{array} \right] \equiv$$

$$\equiv \mathbf{F}\underline{x}(n) + \underline{\tilde{w}}(n) \tag{11}$$

With $\underline{\tilde{w}}(n)$ white gaussian noise, that is, $\underline{\tilde{w}}(n) \sim \mathcal{N}(\underline{0}, \sigma^2 \mathbf{I}_{2NT})$. This last expression has the utility to reflect the contribution of the in-phase and quadrature components of the transmitted symbols into the real and imaginary parts of the received samples in a general form. It is only necessary to compute the matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ derived from the set of constituent matrices $\{\mathbf{A}_q\}$, $\{\mathbf{B}_q\}$, $q = 1...Q$ and estimate the channel matrix $\mathbf{H}$ to then proceed to estimate the symbols $\underline{\alpha}$(n) and $\underline{\beta}$(n).

C. Least Squares Estimation and Decoding

Now, we can use (11) to estimate the components of the transmitted symbols by applying a least squares criterion:

$$\underline{\hat{x}}(n) = \arg \min_{\underline{x}(n)} \|\underline{\tilde{y}}(n) - \mathbf{F}\underline{x}(n)\|^2 \tag{12}$$

which is also a ML criterion under the gaussian assumption of $\underline{\tilde{w}}(n)$. The well-known solution of the pseudo-inverse yields:

$$\underline{\hat{x}}(n) = \left[\begin{array}{c} \underline{\hat{\alpha}}(n) \\ \underline{\hat{\beta}}(n) \end{array} \right] = \mathbf{F}^{\#}\underline{\tilde{y}}(n) \tag{13}$$

$$\mathbf{F}^{\#} = (\mathbf{F}^T\mathbf{F})^{-1}\mathbf{F} \tag{14}$$

Matrix transposition $(.)^T$ is used instead of the hermitian $(.)^H$ since all the elements of $\mathbf{F}$ are real, as can be observed from (11). Then, if we apply (11) and (14) into (13) it results:

$$\left[\begin{array}{c} \underline{\hat{\alpha}}(n) \\ \underline{\hat{\beta}}(n) \end{array} \right] = \left[\begin{array}{c} \underline{\alpha}(n) \\ \underline{\beta}(n) \end{array} \right] + \mathbf{F}^{\#}\underline{\tilde{w}}(n) \tag{15}$$

One key advantage of this way of decoding is that a change in the number of receive antennas, N, only represents a change of the dimensions of $\mathbf{H}$ while the way of constructing $\mathbf{F}$ is the same.

This formulation is code-dependent, but it provides a closed formula to decode any STB code in terms of its structure, defined in its constituent set of $2Q$ matrices. When using quaternary modulations, every component of $\underline{\alpha}(n)$ and $\underline{\beta}(n)$ carries only the information of one bit, so a binary decoding can be done.

IV. Binary Turbo Decoding of Quaternary Modulations

Depending on the number of bits that enter the turbo coder [11] at each considered trellis transition, we talk about binary codes, double-binary codes, or, more generally, b-binary codes. Non-binary codes have the advantage of supporting higher-rate modulations at the cost of exponentially increasing the decoding complexity and also degrading the interleaving gain. When using non-binary codes, the interleaving has to be done at symbol level (not at bit level) in order to guarantee the correct alignment of the parity. With a fixed frame length of N_{TC} bits, a symbol-level interleaver would have a size of N_{TC}/b (b stands for the number of bits per symbol) while a bit-level interleaver would have a size of N_{TC}. This results in a loss of degrees of freedom in randomizing the output sequence that causes a loss in performance.

We will consider a signal constellation with an alphabet of size $\mathcal{M}$. Then the number of bits per symbol is $b = \log_2(\mathcal{M})$. When b-binary turbo decoding [12], the decoder calculates the following Log-Likelihood Ratio (LLR) for each binary word of size b, $\underline{u}_k$ (see Figure 1):

$$LLR(\underline{u}_k) = \log \frac{P(\underline{u}_k|\underline{r}_k)}{P(\underline{u}_k = \underline{0}|\underline{r}_k)} \tag{16}$$

where $\underline{r}_k$ stands for the received sequence (the inputs of the turbo decoder). Expression (16) implies the calculation of $(2^b - 1)$ different ratios for each word. Then, the decision rule is:

$$\underline{\hat{u}}_k = \left\{ \begin{array}{ll} \underline{0} \quad \text{if } LLR(\underline{u}_k) < 0 & \forall \underline{u}_k \neq \underline{0} \\ \arg\max_{\underline{u}_k}\{LLR(\underline{u}_k)\} & \text{otherwise} \end{array} \right. \tag{17}$$

We apply the max-Log-MAP algorithm [13] to compute (16), which is a reduced-complexity version of the optimal MAP decoding [14]. Then, the calculation of the LLR's is done in the logarithmic domain using an iterative routine. The LLR can be expressed as:

$$\begin{aligned} LLR(\underline{u}_k) \;&=\; \max_s\{A_{k-1}(s') + \Gamma_k^{\underline{u}}(s',s) + B_k(s)\} - \\ &-\; \max_s\{A_{k-1}(s') + \Gamma_k^{\underline{0}}(s',s) + B_k(s)\} \end{aligned} \tag{18}$$

where the index k refers to the position of the word over the total of words in the same frame and s and s' denote states of the trellis at a given step. $\Gamma_k^{\underline{u}}(s',s)$ is a metric associated with the trellis transition $(s' \rightarrow s)$ when the word $\underline{u}$ enters the coder and does not have to be updated at each iteration. $A_{k-1}(s')$ is a measure of the probability of being at state s' at the time in which the word $\underline{u}_k$ entered the coder (taking into account the words before it). $B_k(s)$ is a measure of the probability of leaving the trellis in the state s after $\underline{u}_k$ (taking into account the words after it). The values of $A_{k-1}(s')$, $B_k(s)$ can be rapidly calculated recursively, whereas the calculation of $\Gamma_k^{\underline{u}}(s', s)$ depends on the type of channel and noise.

The most complex part of the algorithm is the $\max\{.\}$ search, which has to be done twice each iteration to calculate $(2^b - 1)$ different LLR's for each word. This exponential increase in complexity (and also the degradation of the interleaving gain) can be avoided if bit-level estimates can be passed to the decoder. The way of obtaining this estimates hardly depends on the modulation used, but we will concentrate on quaternary modulations.

As we have found a way to obtain bit level estimates when using quaternary modulations in Section II.C, binary decoding can be applied. This is because in quaternary modulations, each symbol carries the information relative to 2 bits and it is enough to estimate its in-phase and quadrature components to split this information into two binary estimates. In our case, a QPSK channel is split into two parallel BPSK channels. Then, the low-complexity binary max-Log-MAP algorithm can be used to decode the bits taking as inputs the rearranged $\underline{\hat{\alpha}}$(n) and $\underline{\hat{\beta}}$(n).

V. Performance Evaluation

The concatenation of a turbo code and a STB code applying the binary turbo decoding while using QPSK modulation is simulated with two different STB codes. In the first configuration, the STB code used was *Alamouti*'s [15] one:

$$\mathcal{G}_1 = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \tag{19}$$

In the second one, the STB code used was taken from [16]:

$$\mathcal{G}_2 = \begin{bmatrix} s_3 & 0 & s_2 & s_1 \\ 0 & s_3 & s_1^* & -s_2^* \\ s_2^* & s_1 & -s_3^* & 0 \\ s_1^* & -s_2 & 0 & -s_3^* \end{bmatrix} \tag{20}$$

As these two codes transmit the same energy in each channel use, the proposed normalization algorithm in Appendix I will only represent a global scaling factor of all the constituent matrices. However, when codes with a more sophisticated structure have to be used, the scaling factor varies row-by-row and is different for each matrix. One example of these kind of STB codes is [5]:

$$\mathcal{G}_3 = \begin{bmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} \\ -s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} \\ \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{(-s_1-s_1^*+s_2-s_2^*)}{2} \\ \frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & \frac{(s_2+s_2^*+s_1-s_1^*)}{2} \end{bmatrix} \tag{21}$$

Performance results will be given for $\mathcal{G}_1$ and $\mathcal{G}_2$ considering two different Rayleigh fading conditions for each code: 1) *quasi-static*, which means that the MIMO channel matrix $\mathbf{H}$ is constant within all the space-time blocks pertaining to the same frame of N_{TC} bits and independent from frame to frame; 2) *fully-diversity*, where $\mathbf{H}$ varies from block to block. Perfect Channel State Information (CSI) knowledge at the receiver will be considered.

The turbo code used in both schemes is the same. Table I summarizes its parameters, where the inner interleaver belongs to the S-random family described in [17].

TABLE I
Parameters of the Turbo Code used in the Simulation

Frame Size	N_{TC} = 400 bits
RSC's polynomial generator	[7,5]
Rate	1/3 (no puncturing)
Inner Interleaver	S-random S=13
Decoding Algorithm	max-Log-MAP
Decoding Iterations	4

The performance of both configurations (concatenation of the turbo code described above and the STB codes $\mathcal{G}_1$ and $\mathcal{G}_2$) is depicted in Figures 2 and 3. Figure 2 is concerned with the BER in the quasi-static fading channel case, while Figure 3 shows the BER performance in a fully-diversity channel.

Two aspects derived from the curves have to be remarked:

- In both cases, the slope of the curves is greater in the fully-diversity case than in the quasi-static. This is because in the later, the channel is richer in diversity since each block sees a different $\mathbf{H}$. Then, bits which encountered good fading coefficients have error-correcting capacity over bits that suffered severe fading in the same frame, due to the inner interleaver of the turbo structure.
- Specially in the quasi-static channel $\mathcal{G}_2$ performs better than $\mathcal{G}_1$, because the former has rate $3/4$ and the latter 1. Since $\mathcal{G}_2$ is not a full-rate orthogonal complex design it offers more diversity per symbol than $\mathcal{G}_1$ which is full-rate. In fact, full-rate complex orthogonal designs doesn't exist for $M > 2$ [5].

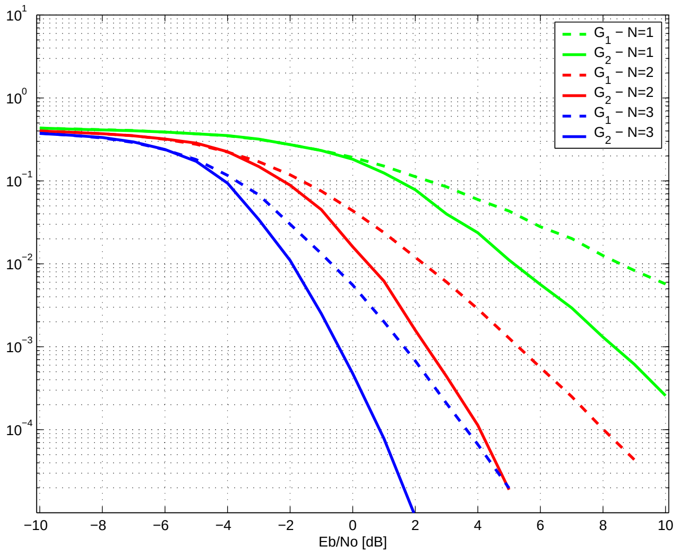

Fig. 2. BER curves of $\mathcal{G}_1$ and $\mathcal{G}_2$ in a quasi-static fading channel

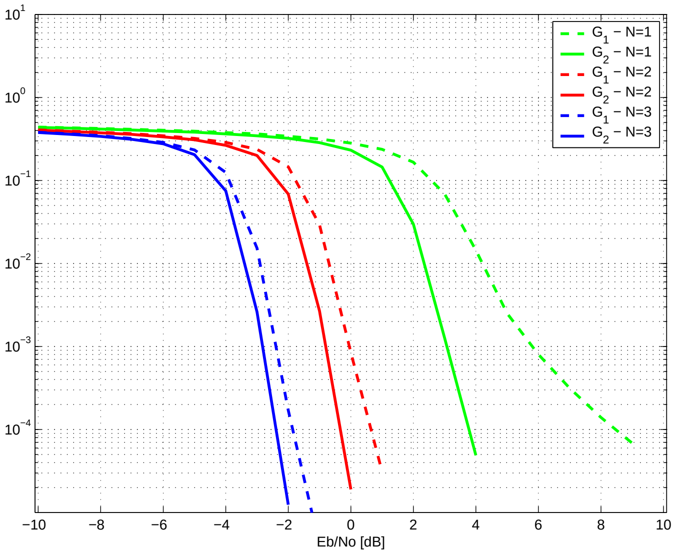

Fig. 3. BER curves of $\mathcal{G}_1$ and $\mathcal{G}_2$ in a fully-diversity fading channel

VI. Conclusions

In this work, a general matrix formulation for the reception of STB codes have been presented. This formulation is very flexible and allows STB decoding for every arbitrary number of receive antennas. In addition, estimates of the in-phase and quadrature components of the symbols can be done. Then, the QPSK modulation can be divided into two BPSK channels. This rearranged estimates can be used by the turbo decoder to start the decoding process. As a result, a way of extending binary turbo decoding while using quaternary modulations has been provided. The key advantage of this system with respect to a double-binary decoding is an increased rate at no complexity penalty. Future work will address the extension of binary turbo decoding to higher-order modulations and an explicit comparison with a b-binary turbo decoding scheme.

Appendix I
Energy Normalization of STB Codes

Denoting by E_s the energy of one transmitted symbol and $\mathcal{E}(.)$ as the statistical expectation operator, we want to normalize the set of matrices $\{\mathbf{A}_q\}$, $\{\mathbf{B}_q\}$, $q = 1...Q$ to achieve:

- $\mathcal{E}\{trace(\mathbf{S}^H(n)\mathbf{S}(n))\} = QE_s$ - The total energy transmitted during a space-time block has to be equal to the energy of Q symbols, forcing the system to not increase the effective energy per symbol.
- $\mathcal{E}\{|\underline{s}_i(n)|^2\} = QE_s/T$, $i = 1...T$ - The same energy has to be transmitted per channel use. This condition forces the STB code to perform in energy terms as a single-antenna system transmitting one unique symbol per channel use.

Since $\mathcal{E}\{trace(\mathbf{S}^H(n)\mathbf{S}(n))\} = \sum_{i=1}^{T}\mathcal{E}\{|\underline{s}_i(n)|^2\}$ (recall (2)), if we guarantee the second condition then the first will also be accomplished. It will be assumed that

$$\mathcal{E}\{\alpha_q\} = \mathcal{E}\{\beta_q\} = 0 \tag{22}$$

$$\mathcal{E}\{\alpha_m\alpha_n\} = \mathcal{E}\{\beta_m\beta_n\} = \mathcal{E}\{\alpha_m\beta_n\} = \frac{E_s}{2}\delta[m-n] \tag{23}$$

that is, the signal constellation is symmetrical with respect to the center of axis, all the symbols have equal probability and are independent of each other. $\delta[m]$ stands for the Kronecker's delta. Decomposing the set of constituent matrices row by row we have:

$$\mathbf{A}_q = [\underline{a}_1^q(n) \quad \underline{a}_2^q(n) \quad ... \quad \underline{a}_T^q(n)]$$
$$\mathbf{B}_q = [\underline{b}_1^q(n) \quad \underline{b}_2^q(n) \quad ... \quad \underline{b}_T^q(n)]$$

for $q = 1...Q$. Then applying (22), (23) and the later decomposition, it yields:

$$\begin{aligned}\mathcal{E}\{|\underline{s}_i(n)|^2\} &= \mathcal{E}\{|\sum_{q=1}^{Q}(\alpha_q\underline{a}_i^q + j\beta_q\underline{b}_i^q|^2\} = \\ &= \frac{E_s}{2}\sum_{q=1}^{Q}(|\underline{a}_i^q|^2 + |\underline{b}_i^q|^2)\end{aligned} \tag{24}$$

Then, using (24) in the second condition it becomes:

$$\sum_{q=1}^{Q}(|\underline{a}_i^q|^2 + |\underline{b}_i^q|^2) = \frac{2Q}{T} \quad i = 1...T \tag{25}$$

which forces a row by row normalization of the constituent matrices of the STB code, defining the normalization matrix $\mathbf{\Gamma}$ as follows:

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_1 & 0 & \cdots & & 0 \\ 0 & \gamma_2 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ & & & & 0 \\ 0 & \cdots & & 0 & \gamma_T \end{bmatrix} \in \mathbb{R}^{T\times T} \tag{26}$$

$$\gamma_i \equiv \left(\frac{2Q/T}{\lambda_i}\right)^{1/2} \quad i = 1...T \tag{27}$$

$$\lambda_i \equiv \sum_{q=1}^{Q}(|\underline{a}_i^q|^2 + |\underline{b}_i^q|^2) \quad i = 1...T \tag{28}$$

Then, the normalization of the STB code to make a fair energy comparison with single-antenna systems can be done with the matrix product:

$$\mathbf{A}_q^{norm} = \mathbf{A}_q\mathbf{\Gamma} \quad q = 1...Q \tag{29}$$
$$\mathbf{B}_q^{norm} = \mathbf{B}_q\mathbf{\Gamma} \quad q = 1...Q \tag{30}$$

Finally, the set of constituent matrices $\{\mathbf{A}_q^{norm}\}$, $\{\mathbf{B}_q^{norm}\}$, $q = 1...Q$ is used instead of the original one.

Acknowledgment

The author would like to thank J. A. López-Salcedo for his encouragement, valuable comments and critical feedback.

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